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Your Roll No. :2019.....

Sl. No. of Q. Paper : **7465** **J**

Unique Paper Code : 32351303

Name of the Course : **B.Sc.(Hons.)**
Mathematics

Name of the Paper : Multivariate Calculus

Semester : III

Time : 3 Hours **Maximum Marks : 75**

Instructions for Candidates :

- (i) Write your Roll No. on the top immediately on receipt of this question paper.
- (ii) **All** Sections are compulsory.
- (iii) Attempt any **five** questions from each **Section**.
- (iv) All questions carry equal marks.

Section- I

1. Given that the function

$$f(x, y) = \begin{cases} \frac{3x^3 - 3y^3}{x^2 - y^2} & \text{for } x^2 \neq y^2 \\ B & \text{otherwise} \end{cases}$$

is continuous at the origin, what is B ?

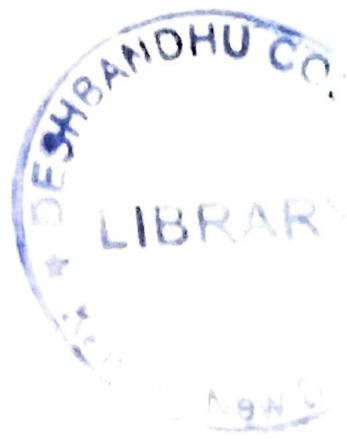
2. In physics, the *wave equation* is :

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is :

$$\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Determine whether $z = \sin 5ct \cos 5x$ satisfies the wave equation, the heat equation, or neither.



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3. The radius and height of a right circular cone are measured with errors of at most 3% and 2%, respectively. Use increments to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.
4. If $f(x, y, z) = xyz^2 e^{xz}$ and $x = 2 + 3t$, $y = 6 - 4t$, $z = t^2$. Compute $\frac{df}{dt}(1)$.
5. Sketch the level curve corresponding to $C = 1$ for the function $f(x, y) = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ and find a unit normal vector at the point $P_0(2\sqrt{3})$.
6. Find the point on the plane $2x + y - z = 5$ that is closest to the origin.

Section - II

- 7.** Find the volume of the solid bounded above by the plane $z = y$ and below in the xy -plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.
- 8.** Sketch the region of integration and then compute the integral $\int_0^1 \int_x^{2x} e^{y-x} dy dx$ in 2 ways :
- (a) with the given order of integration
- (b) with the order of integration reversed
- 9.** Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2 + y^2} dy dx$ by converting to polar coordinates.
- 10.** Find the volume of the tetrahedron bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0$, $y = 0$ and $z = 0$.



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11. Compute $\iiint_D \frac{dx dy dz}{\sqrt{x^2 + y^2 + z^2}}$ where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

12. Use the change of variables to compute

$$\iint_D \frac{(x-y)^4}{(x+y)^4} dy dx, \text{ where D is the triangular}$$

region bounded by the line $x + y = 1$ and the coordinate axes.

Section - III

13. Find the work done by the force field

$$\vec{F} = \frac{x}{\sqrt{x^2 + y^2}} \vec{i} - \frac{y}{\sqrt{x^2 + y^2}} \vec{j} \text{ when an object moves}$$

from $(a,0)$ to $(0,a)$ on the path $x^2 + y^2 = a^2$.

14. Verify that the following line integral is

independent of the path $\oint (3x^2 + 2xy^2) dx + (2xy + y^3) dy$ where C is any path from $(0,0)$ to $(0,1)$.

15. Use Green's theorem to evaluate $\oint_C (x \sin x dx - \exp(y^2) dy)$ where C is the closed curve joining the points $(1, -1)$, $(2, 5)$ and $(-1, -1)$ in counterclockwise direction.

16. State Stoke's theorem and use it to evaluate $\iint_S \text{curl} \vec{F} \cdot d\vec{S}$ where $\vec{F} = xz\vec{i} + yz\vec{j} + xy\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane.

17. Use the divergence theorem to evaluate the surface integral $\iint_S \vec{F} \cdot \vec{N} dS$, where $\vec{F} = (x^2 + y^2 - z^2)\vec{i} + yx^2\vec{j} + 3z\vec{k}$; S is the surface comprised of the five faces of the unit cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$, missing $z = 0$.



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18. Evaluate $\iint_S 2x \, dS$ where S is the portion of the plane $x + y + z = 1$ with $x \geq 0, y \geq 0, z \geq 0$.

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